# The effects of loading variables on overload induced fatigue crack growth retardation parameters

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A growing fatigue crack is known to be retarded on application of an overload cycle. The retardation may be characterized by the total number of cycles involved during retardation,  $N_{\rm D}$  and the retarded crack length,  $a_{\rm D}$ . The loading variables play an important role to influence the retardation behaviour and thereby the retardation parameters  $N_{\rm D}$  and  $a_{\rm D}$ . The important loading variables are considered to be the stress intensity range,  $\Delta K$  during constant amplitude loading, stress ratio, R and the ratio of the overload stress to the maximum stress in constant amplitude loading, OLR etc. The objective of the present investigation is to study the effect of different loading variables on the retardation parameters. The investigation has been conducted in a 2024-T3 Al-alloy with centre crack tension panels loaded using an Electromagnetic Resonance Machine. The crack extension and crack closure have been measured during cyclic loading under constant amplitude and under overloading situations. The results do indicate dependence of crack opening stress intensity factor,  $K_{OP}$  on loading variables such as  $\Delta K$ , R and OLR. The delay cycle  $N_D$  is found to increase with loading variables  $\Delta K$ , OLR and R. The functional dependence of retardation zone a<sub>D</sub> on loading variables has also been identified. © 1999 Kluwer Academic Publishers

# Nomenclature

а	Crack length
$\Delta a$	Crack extension
$a_{\rm D}$	Retarded crack length
Ν	Number of fatigue cycles
$N_{\rm D}$	Number of delay cycles due to overload
K <sub>max</sub>	Maximum stress intensity factor
$K_{\min}$	Minimum stress intensity factor
$\Delta K$	$K_{\rm max} - K_{\rm min}$ stress intensity range
$K^{OL}$	K corresponding to overload
K <sub>OP</sub>	Crack opening stress intensity factor
$K_{\rm OP}^{\rm OL}$	Crack opening stress intensity factor after
01	overload application
$\Delta K^{\rm OL}$	$(K^{OL} - K_{min})$ range of K for overloading
	cycle
R	Stress ratio (min to max stress in constant
	amplitude loading)
OLR	Overloading ratio (ratio of overload stress
	to max. stress in a constant amplitude
	loading cycle)
В	Thickness
$U^{OL}$	$(K^{\rm OL} - K_{\rm OP})/\Delta K$ , dimensionless effective
	stress intensity range in overload condition
$r_{\rm c}^{\rm OL}$	Overload induced cyclic plastic zone radius

# 1. Introduction

Many engineering components and structures experience occasional overloads during service. Such overloads lead to retardation of a growing fatigue crack which may even culminate into crack arrest. Further, in fatigue critical structures, fatigue crack propagation under service conditions generally involves random or variable amplitude, rather than constant amplitude loading, such that the loading history i.e. interaction between loading cycles, can be a major factor determining the fatigue life. Single overload cycle is the primitive building block of the variable amplitude loading and represents the simplest situation involving retardation. Thus, the response of the material to single overload cycle should be understood for more versatile modeling of random load fatigue. The effect of overload has been experimentally observed by application of single overload cycle over a constant amplitude fatigue test [1, 2]. The overloading effect has been schematically represented in Fig. 1 in the form of crack length, a vs. number of cycle, N plot. The overload introduces a large plastic zone and hence residual compressive stresses in the vicinity of crack tip [3, 4]. The residual compressive stresses result in closure over a length longer than that during



No of stress cycle, N

Figure 1 Schematic representation of the response of fatigue crack growth on overload application.

the constant amplitude condition [3, 5]. As a result a fatigue cycling can cause a growth only if the enhanced residual stresses are overcome to a degree that the crack tip is open again. This explains the low growth rate of the crack on tensile overload application. The effect of loading condition on crack closure and its role on the post overload retardation behaviour has been studied by several investigators [2, 5, 6]. However, except a few cases [2, 6], no quantitative correlation has been reported in literature connecting overload induced closure parameter with various loading variables (i.e.  $\Delta K$ ,  $K^{OL}$ , OLR etc).

The extent of retardation is usually expressed in terms of retarded crack length,  $a_D$  (see Fig. 1). It is observed by a few investigators that the overload affected crack length,  $a_D$  is commonly associated with the monotonic plastic zone at the crack tip [7, 8]. However, no systematic study has been reported correlating this retardation parameter with the applied loading variables.

The delay cycle,  $N_D$  as represented in Fig. 1 is the number of load cycles required to achieve the preoverload crack growth rate. The  $N_D$  is considered to be an important parameter to characterize the crack growth retardation on overload application. Therefore an attempt has also been made to study the effect of basic stress intensity factor range,  $\Delta K$  and OLR on delay cycle,  $N_D$ .

### 2. Experimental programme

The experimental study was conducted on a standard 2024-T3 Aluminium alloy of yield strength 377 MPa. The above material finds wide application in aircraft structures in the condition investigated. The material was available in sheet form of thickness 3.38 mm. The centre cracked tension (CCT) geometry with dimension of  $220.00 \times 85.00 \times 3.18$  mm were used for the fatigue tests. The fatigue tests were carried out in tension-tension mode under constant stress amplitude sinusoidal loading at stress ratios 0.17, 0.30 and 0.44 and at a frequency of 97 Hz, using an Electromagnetic Resonance Machine. The crack growth measurement was done using Electrical Potential Difference Technique. To measure the opening/closing load miniature clip gauge of 4.0 mm gauge length was mounted at

a distance of 1 mm behind the crack tip and loading frequency for the purpose was maintained at 0.05 Hz. The specimens were subjected to overloading cycle at appropriate crack lengths. The level of overloading ratio, OLR (=  $K_{\text{overload}}/K_{\text{max}}$ ) applied to the specimens were 1.5, 1.7, 1.9 and 2.1. The crack opening and closing loads were measured at various crack lengths for both the constant amplitude and overload conditions of specimens.

### 3. Results and discussion

The effect of introducing an overload spike over a constant amplitude cyclic loading are presented in the form of crack extension  $\Delta a$  vs. N plot in Fig. 2. It is observed that the application of an overload cycle results in crack growth retardation and the magnitude of retardation increases with increasing overloading ratio, OLR.

## 3.1. Effect of $\Delta K$ on crack closure

The variation of overload induced crack closure stress intensity factor (SIF),  $K_{OP}^{OL}$ , a retardation parameter with applied stress intensity range,  $\Delta K$  has been presented in Fig. 3 on semi-log scale. The plot shows that  $K_{OP}^{OL}$  increases with increase in  $\Delta K$  and OLR level for the investigated range of  $\Delta K$  (7–13 MPa  $\sqrt{m}$ ). Superimposed on the diagram are the  $K_{OP}$  values obtained under constant amplitude loading condition. Linear relationship may be noticed between  $K_{OP}^{OL}$  and  $\Delta K$  at constant values of OLR and *R* which conform to the following equations,

$$\log K_{\rm OP}^{\rm OL} = (6.3 \,\Delta K + 26.8) 10^{-2}$$

$$R = 0.17 \text{ and } OLR = 1.9$$
(1)

$$\log K_{\rm OP}^{\rm OL} = (10.3 \,\Delta K - 24.0) 10^{-2}$$

$$R = 0.17 \text{ and } OLR = 1.0$$
(2)

(constant amplitude loading condition)

$$\log K_{\rm OP}^{\rm OL} = (5.9 \,\Delta K + 37.5) 10^{-2}$$

$$R = 0.30 \quad \text{and} \quad \text{OLR} = 1.9$$
(3)

A comparison of Equation 1 with 2 does reveal that the  $\Delta K$  dependence of  $K_{\text{OP}}^{\text{OL}}$  diminishes with application of



Figure 2 Typical overload cycle modified crack extension vs. no. of stress cycles.



Figure 3 Variation of  $K_{OP}^{OL}$  as a function of applied stress intensity range,  $\Delta K$  and OLR at stress ratio, (a) R = 0.17 and (b) R = 0.30.

overload, as compared to the constant amplitude loading situation. However, the intercept level in the above equations is highest for the case of overloading indicating that the higher value of  $K_{OP}^{OL}$  (as compared to  $K_{\text{OP}}$ , under constant amplitude loading condition) at a given  $\Delta K$  is mainly attributed to this intercept effect. The higher 'intercept effect' is apparently caused due to higher  $K^{OL}$  level in the overloading cycle. The effect of increasing R also leads to a decreasing dependence on  $\Delta K$  and an enhanced intercept effect.

**3.2.**  $K_{OP}^{OL}$  as a function of  $K^{OL}$ An increase in *R* or  $\Delta K$  at a given overloading ratio causes an enhancement in  $K^{OL}$  level. Similarly with increasing OLR, the  $K^{OL}$  increases. Therefore, it approximately the formula of  $K_{OL}^{OL}$  mith OLD. A K or *R* pears that the rising trend of  $K_{\text{OP}}^{\text{OL}}$  with OLR,  $\Delta K$  or R could be a result of increase in  $K^{\text{OL}}$  and corresponding intensification of residual compressive stresses in the plastic zone [9]. Therefore an attempt is made to correlate  $K_{\text{OP}}^{\text{OL}}$  with  $K^{\text{OL}}$  in Fig. 4. It is interesting to note



Figure 4 Effect of overloading stress intensity factor on overload induced crack closure parameter,  $K_{OP}^{OL}$ .

that a unique relationship is obtained between  $K_{OP}^{OL}$  and  $K^{OL}$  irrespective of *R*, OLR and  $\Delta K$  level in the range of their values investigated. The  $K_{OP}^{OL}$  vs.  $K^{OL}$  relation may be expressed by following equation,

$$\log K_{\rm OP}^{\rm OL} = (3.2 \, K^{\rm OL} + 15.4) 10^{-2} \tag{4}$$

Above observation indicates  $K^{OL}$  as a prime variable to control overload induced crack closure level.

**3.3.**  $U_{\min}^{OL}$  as a function of  $\Delta K$  and R. The crack closure due to overloading cycle modifies the effective stress intensity range available for crack growth. The dimensionless effective stress intensity range is given in terms of  $U^{OL}$  which is defined as,

$$U^{\rm OL} = \left(K_{\rm max} - K_{\rm OP}^{\rm OL}\right) / (K_{\rm max} - K_{\rm min}) \qquad (5)$$

In the present work,  $U^{OL}$  is obtained corresponding to maximum  $K_{OP}^{OL}$  and denoted as  $U_{\min}^{OL}$ . The behaviour of  $U_{\min}^{OL}$  with  $\Delta K$  is shown in Fig. 5 at two different values of R. A decreasing nature of  $U_{\min}^{OL}$  is noticed with increasing  $\Delta K$ . Whereas  $U_{\min}^{OL}$  is found to increase with R at a given  $\Delta K$  in this alloy, a decrease in  $U_{\min}^{OL}$ with increasing  $\Delta K$  is expected as  $K_{\text{OP}}^{\text{OL}}$  increases with  $\Delta K$  contributing to a reduction in  $U_{\text{min}}^{\text{OL}}$  value. An increase in  $U_{\text{min}}^{\text{OL}}$  with *R* (at a fixed  $\Delta K$ ) is related to a dominating role of *K*. inating role of  $K_{\text{max}}$  over the  $K_{\text{OP}}$ . Though, the latter (i.e.  $K_{OP}$ ) increases with R, the  $K_{max}$  also increases and the overall effect is an increase in term  $(K_{\text{max}} - K_{\text{OP}})$ .

## 3.4. Dependence of $a_D$ on $\Delta K$ and OLR

The variation of  $a_D$  with  $\Delta K$  is shown in Fig. 6. At a constant R and OLR level,  $a_D$  is generally found to increase with  $\Delta K$ . It may be noted that in the above diagram  $a_D$  is simultaneously under the influence of  $\Delta K$  as well as  $K^{OL}$  as the latter also increases with  $\Delta K$ . Therefore the intrinsic effect of  $\Delta K$  on  $a_D$  is not realized from this diagram. Increase in  $a_D$  with basic stress intensity range has also been reported by the other investigators [8].

The effect of OLR on  $a_D$  is presented in Fig. 7. The  $a_{\rm D}$  values are found to increase with OLR in the beginning and beyond a certain level of OLR, a<sub>D</sub> tends to decrease. As R and  $\Delta K$  are fixed for a given line,



*Figure 5* The crack closure parameter,  $U_{\min}^{OL}$  as a function of  $\Delta K$ .



Figure 6 Variation of retarded crack length,  $a_D$  with  $\Delta K$  at stress ratio, R = 0.30.



Figure 7 Effect of OLR on retarded crack length,  $a_{\rm D}$  at stress ratio, R = 0.30.

increasing OLR has the effect of causing an enhancement in  $K^{\text{OL}}$  value. Thus, it is the  $K^{\text{OL}}$  which results in an increase or decrease of  $a_{\text{D}}$  while OLR is being raised. Analysis of data relating  $a_{\text{D}}$  and  $K^{\text{OL}}$  reveal that the drop/saturation in  $a_{\text{D}}$  value takes place at the level of  $B/B_{\text{p.strain}} = 1.85$  where *B* is the specimen thickness and  $B_{\text{p.strain}}$  is the plane strain thickness requirement for the cyclic plastic zone corresponding to  $\Delta K^{\text{OL}}$ . Some data from literature [1, 10] on an Al alloy and steel have been analyzed and presented in Fig. 8 to ascertain the



*Figure 8* Effect of OLR on retarded crack length,  $a_D$  based on data from literature [1, 10].

dependence of  $a_D$  on OLR or  $K^{OL}$ . It is interesting to note that the transition in  $a_D$  vs.  $K^{OL}$  plots appeared at the level of  $B/B_{p,strain}$  ratio of 1.85 in E-36 Steel, at 1.92 in Al-alloy (CT geometry) and 2.00 in Al-alloy (with CCT geometry). A value of 1.62 has been noted from the work of Mingda et al. [11] for aluminium alloy at the transition point. The residual compressive stress field remains confined in the cyclic plastic zone  $2r_c^{OL}$ produced by  $\Delta K^{OL}$ . Therefore it is likely that beyond a critical value of cyclic plastic zone a decreasing condition of crak-tip constraint sets-up. The above factor will affect retardation zone size,  $a_{\rm D}$  leading to a decrease in  $a_{\rm D}$  beyond a specific level of  $B/B_{\rm p.strain}$ . It may be concluded from this analysis that the size of cyclic plastic zone as is produced by  $\Delta K^{OL}$  i.e.  $r_c^{OL}$  vis a vis specimen stress state controls the behaviour of  $a_{\rm D}$ significantly.

## 3.5. Variation of delay cycle with loading variables

The variation of  $N_D$  as function of  $\Delta K$  is shown in Fig. 9. An increase in  $N_D$  is noted with  $\Delta K$  at a fixed OLR. The effect of R on  $N_D$  may also be observed in this figure. It may further be noticed that at given OLR, a higher R (i.e. R = 0.44) exhibits enhanced value of  $N_D$  as compared to lower R (i.e. R = 0.17 and 0.30) level.

The increase in  $N_{\rm D}$  with  $\Delta K$  is supposed to be due to an increasing ratio of plane stress surface region to specimen thickness [12]. This observation is also in agreement with the effect of  $\Delta K$  on  $K_{\rm OP}^{\rm OL}$  and retarded crack length.

The effect of overloading ratio on delay cycles,  $N_D$  is shown in Fig. 10. The  $N_D$  appears to be a sensitive function of OLR and increases significantly with OLR at a constant  $\Delta K$  and R level in the range of OLR investigated. The relationship may be represented as follows,

$$\log N_{\rm D} = 2.12 \,({\rm OLR}) + 0.784$$
(Al-alloy,  $R = 0.17, \,\Delta K = 8.2 \,{\rm MPa} \,\sqrt{\rm m}$ ) (6)

The increasing OLR would increase the level of overload induced closure,  $K_{OP}^{OL}$  which in turn will result in an increase in  $N_D$  value. Almost similar observation has been reported by some other investigators [2, 11].



*Figure 9* The variation of delay cycle  $N_D$  as a function of  $\Delta K$ .



Figure 10 Effect of OLR on delay cycle, N<sub>D</sub>.

From Fig. 10 the effects of  $\Delta K$  and R may also be noticed at a given OLR level. At a given value of OLR, as  $\Delta K$  increases  $N_D$  is found to increase. An increasing  $\Delta K$  results in an increase in  $K^{OL}$  which may ultimately give rise to an enhancement in delay cycle,  $N_D$  analogous to increase in  $a_D$ .

### 4. Conclusions

The conclusions based on the present investigation are reported as follows:

1. The overload induced crack closure increases with applied  $\Delta K$  and OLR. However  $\Delta K$  dependence of  $K_{\text{OP}}^{\text{OL}}$  diminishes with increase in *R* as well as with level of overloading cycle.

2. The  $K_{OP}^{OL}$  varies linearly with applied overload level as follows,

$$\log K_{\rm OP}^{\rm OL} = (3.2 \, K^{\rm OL} + 15.4) 10^{-2}$$

3. The crack closure level is mainly governed by the maximum SIF of the stress cycle.

4. Retarded crack length,  $a_D$  increases with  $K^{OL}$  till the thickness ratio  $B/B_{p.strain}$  (corresponding to  $\Delta K^{OL}$ ) reaches nearly a value of 1.9, indicating the stress state with respect to cyclic plastic zone  $r_c^{OL}$  as the primary factor controlling the behaviour of  $a_D$ .

5. Delay cycle,  $N_D$  increases linearly on semi-log scale with loading variables  $\Delta K$ , OLR and stress ratio R.

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